**Worksheet 6 Memorandum: Trigonometric Functions**

Grade 11 Mathematics

1.   a)

b)

c)

d)

e)

f)

g)

h)

i)

j)

k)

l)

m)

n)

o)
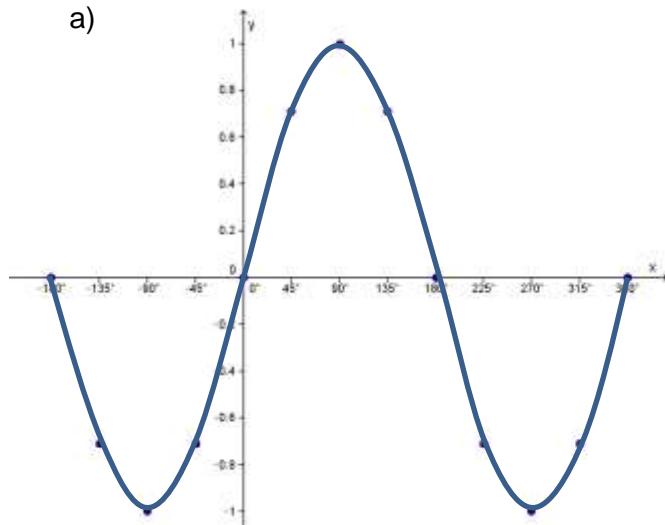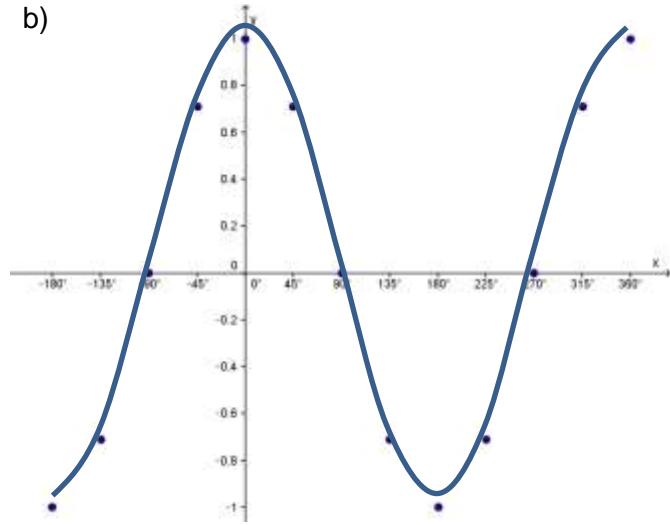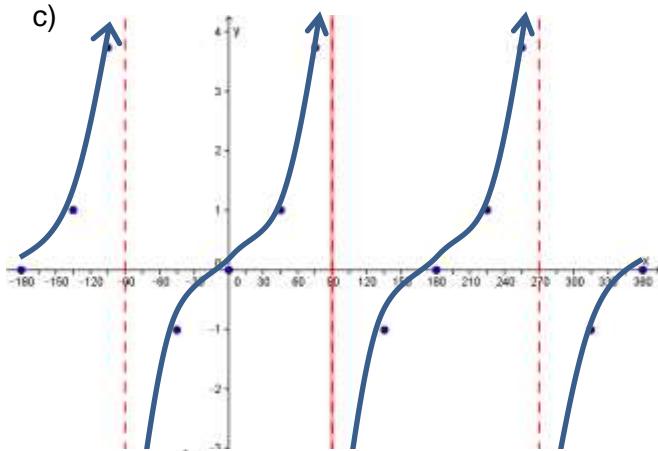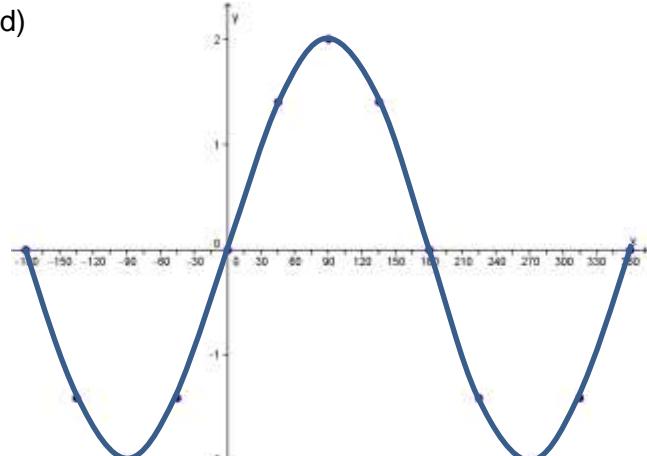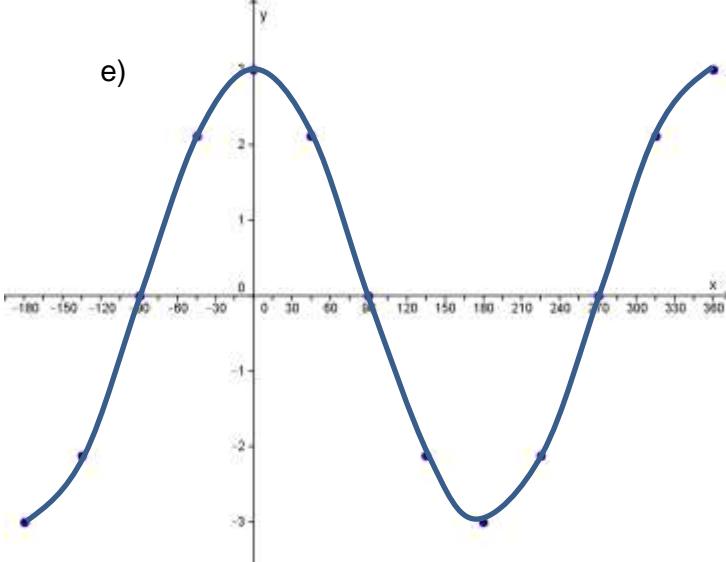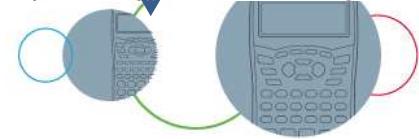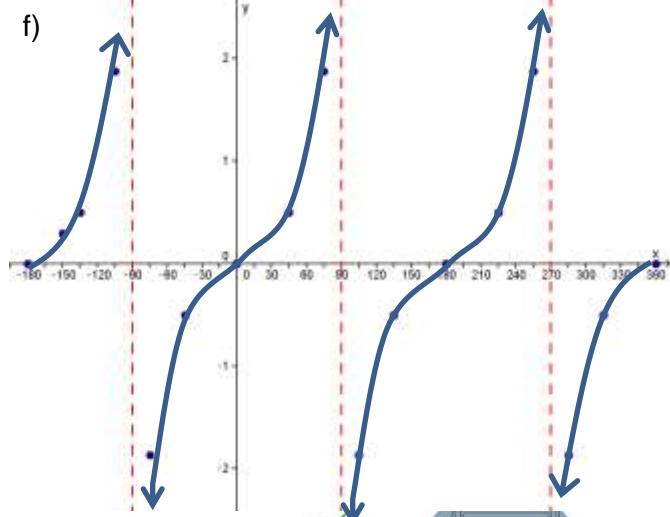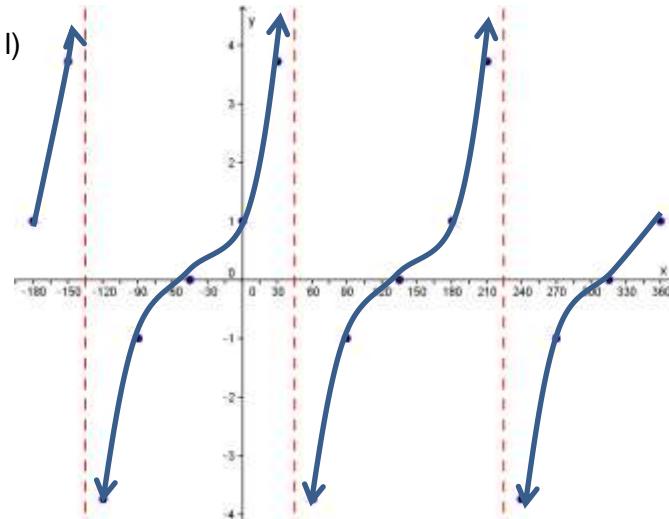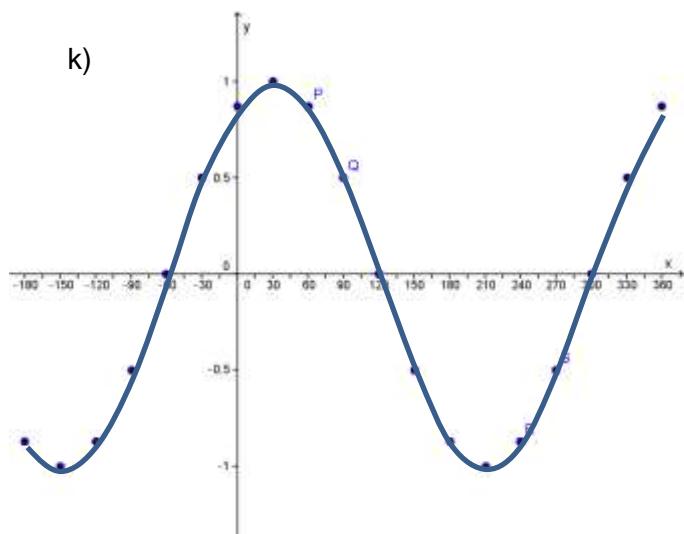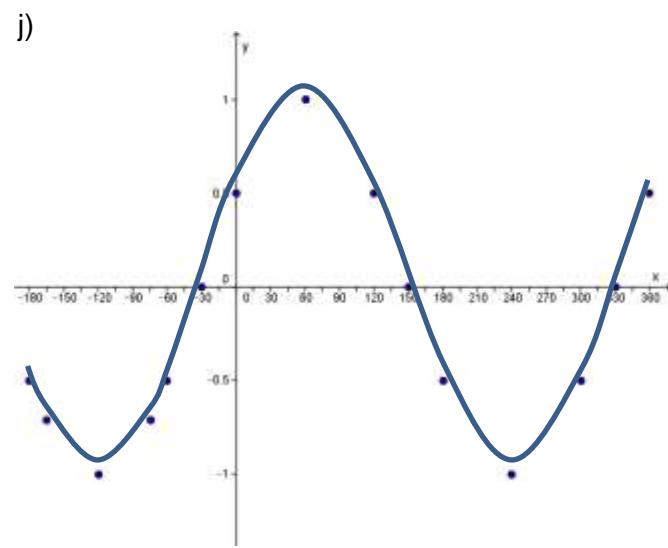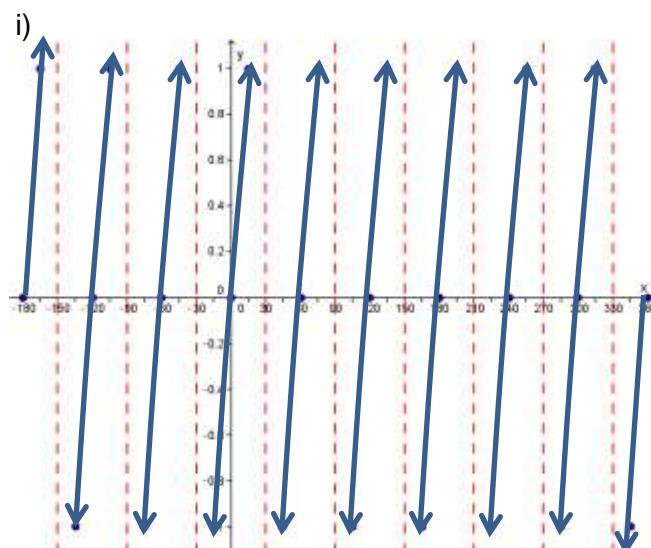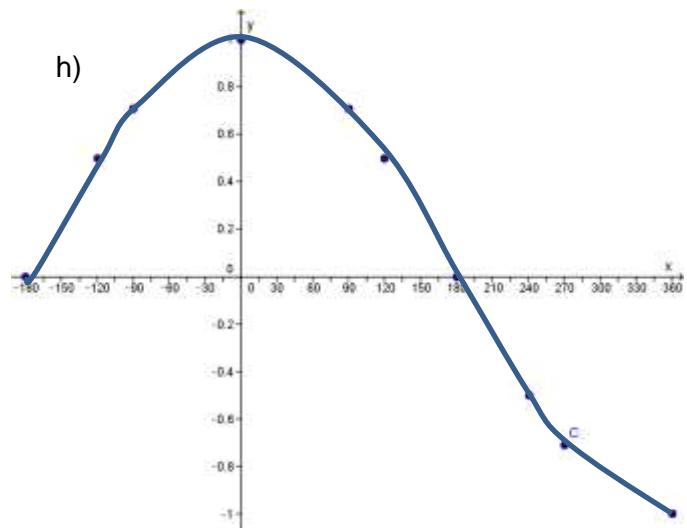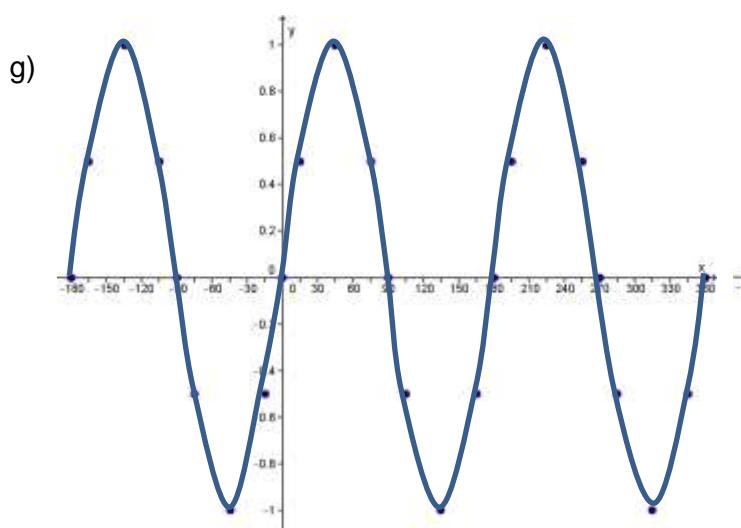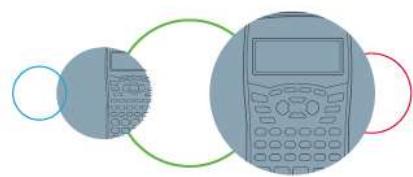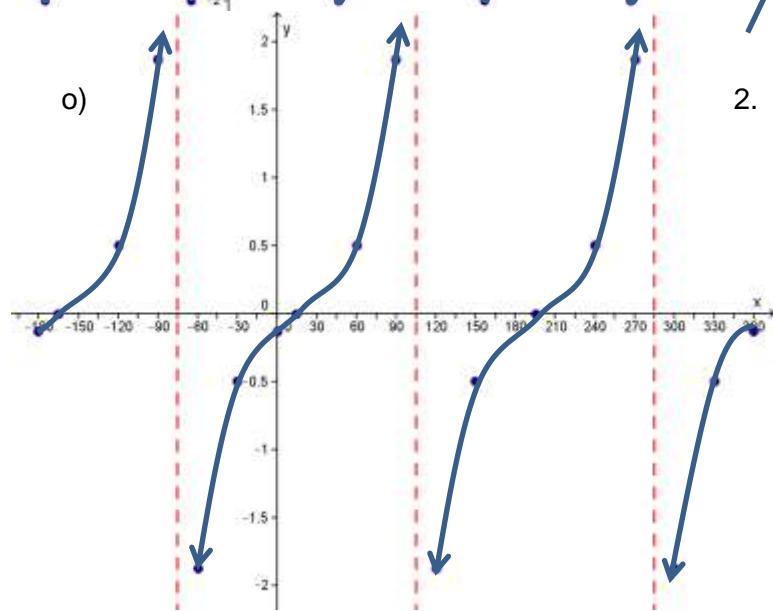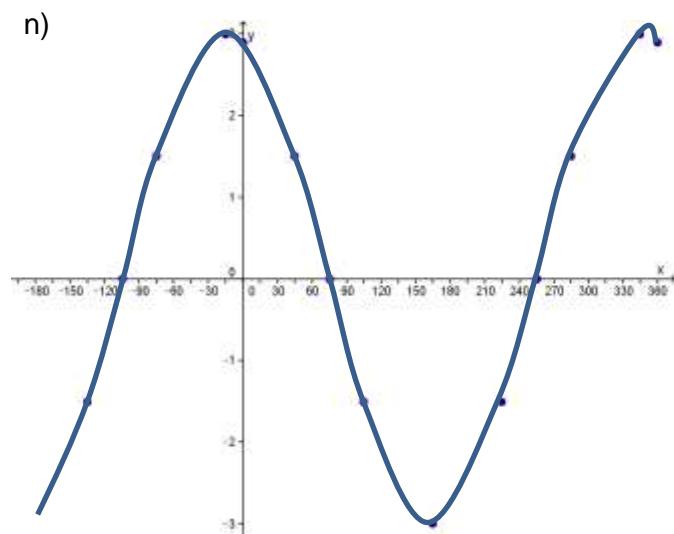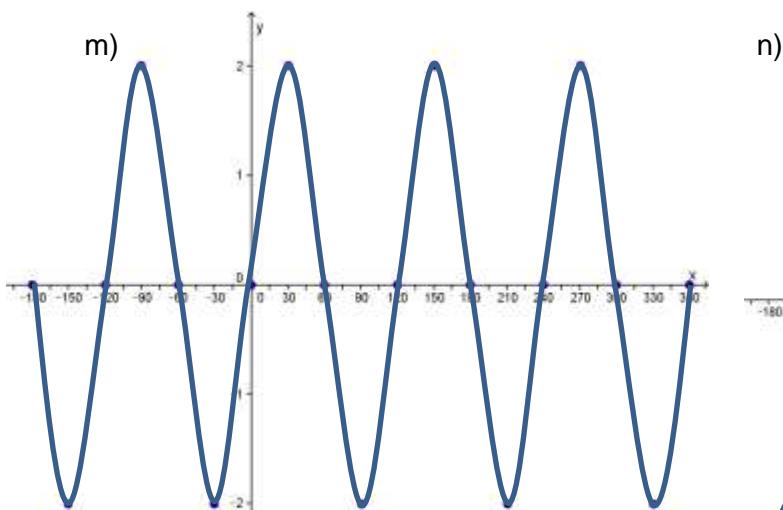
2. a) $y = \sin x$

Period = 360

Amplitude = 1

b) $y = \cos x$

Period = 360

Amplitude = 1

c) $y = \tan x$

Period = 180

Amplitude = no amplitude.

d) $y = 2 \sin x$

Period = 360

Amplitude = 2

e) $y = 3 \cos x$

Period = 360

Amplitude = 3

f) $y = \frac{1}{2} \tan x$

Period = 180

Amplitude = no amplitude

g) $y = \sin 2x$

Period = 180

Amplitude = 1

h) $y = \cos \frac{1}{2} x$

Period = 720

Amplitude = 1

i) $y = \tan 3x$

Period = 60

Amplitude = no amplitude.

j) $y = \sin(x + 30)$

Period = 360

Amplitude = 1

k) $y = \cos(x - 30)$

Period = 360

Amplitude = 1

l) $y = \tan(x + 45)$

Period = 180

Amplitude = no amplitude.

m) $y = 2 \sin 3x$

Period = 120

Amplitude = 2

n) $y = 3\cos(x + 15)$

Period = 360

Amplitude = 3

o) $y = \frac{1}{2} \tan(x - 15)$

Period = 180

Amplitude = no amplitude.
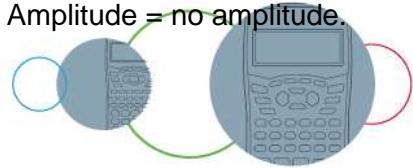
3. a) $y = \sin 2x$ b) $y = 2\cos\frac{1}{2}x$

     c) $y = \tan(x - 15)$ d) $y = 2\cos(x - 60)$ OR $y = 2\sin(x + 30)$

     e) $y = \cos(x - 45)$ OR $y = \sin(x + 45)$

4. a) $y = \sin 2x$ b) $y = 2\cos\frac{1}{2}x$

       Period = 180   Period = 720

       Amplitude = 1   Amplitude = 2

     c) $y = \tan(x - 15)$ d) $y = 2\cos(x - 60)$ OR $y = 2\sin(x + 30)$

       Period = 180   Period = 360   or 360

       Amplitude = no amplitude.   Amplitude = 2   or   2

     e) $y = \cos(x - 45)$ OR $y = \sin(x + 45)$

       Period = 360   or   360

       Amplitude = 1   or   1

5. a) $f(x) = a\sin(x - 15)$ Subs in (-75; -0.5)

       $\therefore -0.5 = a\sin(-75 - 15)$ $g(x)$ is the cos graph shifted 45 degrees left.

       $\therefore -0.5 = a\sin(-90)$ $\therefore b = 45$

       $\therefore a = \frac{-0.5}{\sin(-90)}$

       $\therefore a = \frac{1}{2}$

     b) $f(x) = \frac{1}{2}\sin(x - 15)$ and $g(x) = \cos 2(x + 45)$

       Period = 360   Period = 180

       Amplitude = $\frac{1}{2}$ Amplitude = 1

     c) In other words where $f(x) = g(x)$

       At approximately $x = -103; x = 3; x = 105; x = 175$

     d) $-180 < x \le -103$ and $3 \le x \le 105$ and $175 \le x < 180$

     e) y-intercepts:

       $f(x) = \frac{1}{2}\sin(x - 15) = 0$ AND $g(x) = \cos 2(x + 45) = 0$

       $\therefore \sin(x - 15) = 0$ $\therefore \cos 2(x + 45) = 0$

       $\therefore x - 15 = 0$ $\therefore 2(x + 45) = 90 + k180$

       $\therefore x = 15 + k180$ $\therefore x + 45 = 45 + k90$
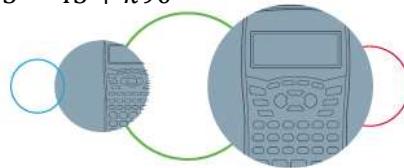
$\therefore x = -165 \; and \; 15$ $\therefore x = 0 + k90$

$(-165; 0) \; and \; (15; 0)$ $\therefore x = -180 \; or -90 \; or \; 0 \; or \; 90 \; or \; 180$

x-intercepts: $(-180; 0) \; (-90; 0) \; (90, 0) \; and \; (180; 0)$

$f(x) = \frac{1}{2}\sin(0 - 15)$ AND $g(x) = \cos 2(0 + 45)$

$\therefore f(x) = \frac{1}{2}\sin(-15)$ $\therefore g(x) = \cos 2(45)$

$\therefore f(x) = \frac{1}{2}\left(\frac{-\sqrt{6}+\sqrt{2}}{4}\right)$ $\therefore g(x) = \cos 90$

$\therefore f(x) = -0.129$ $\therefore g(x) = 0$

$(0; -0.129)$ $(0; 0)$

6.   a) $m(x) = p(x)$

           $x = -150$ OR $x = -90$ OR $x = 90$ OR $x = 330$

   b) $x = -150$ OR $x = -90$ OR $x = 90$ OR $x = 330$

           When the distance is at a minimum the graphs intersect, thus the answer is the same as the previous questions answer.

     c) $m(x) = 2\sin\frac{1}{2}x$ AND $p(x) = 2\cos(x - 45)$

           Period = 720   Period = 360

           Amplitude = 2   Amplitude = 2

   d)   y-intercepts:

           $m(x) = 2\sin\frac{1}{2}(0)$ AND $p(x) = 2\cos(0 - 45)$

           $\therefore m(x) = 2(0)$ $p(x) = 2(0.707)$

           $\therefore m(x) = 0$ $p(x) = 1.414$

           $(0; 0)$ $(0; -1.414)$

           $x -$ intercepts:

           $m(x) = 2\sin\frac{1}{2}x = 0$ AND $p(x) = 2\cos(x - 45) = 0$

           $\therefore \sin\frac{1}{2}x = 0$ $\therefore \cos(x - 45) = 0$

           $\therefore \frac{1}{2}x = 0$ $\therefore x - 45 = 90$

           $\therefore x = 0 \; or -360 \; or \; 360$ $\therefore x = 135 \; or \; -45 \; or \; -225 \; or -360 \; or \; 315$

$(-360; 0) \; or \; (0; 0) \; or \; (360; 0)$ $(135; 0) or \; (-45; 0) \; or \; (-225; 0) \; or \; (-360; 0) \; or \; (315; 0)$

     e) $Distance = m(x) - p(x)$ $at \; x = 225$

               $= 2\sin\frac{1}{2}(225) - 2\cos(225 - 45)$

               $= 1.848 - (-2)$

               $= 3.848$
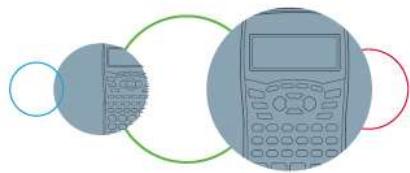
7. a) $h(x) = \sin(x - 15)$, $j(x) = \cos(x - 15)$ and $k(x) = \tan(x - 15)$
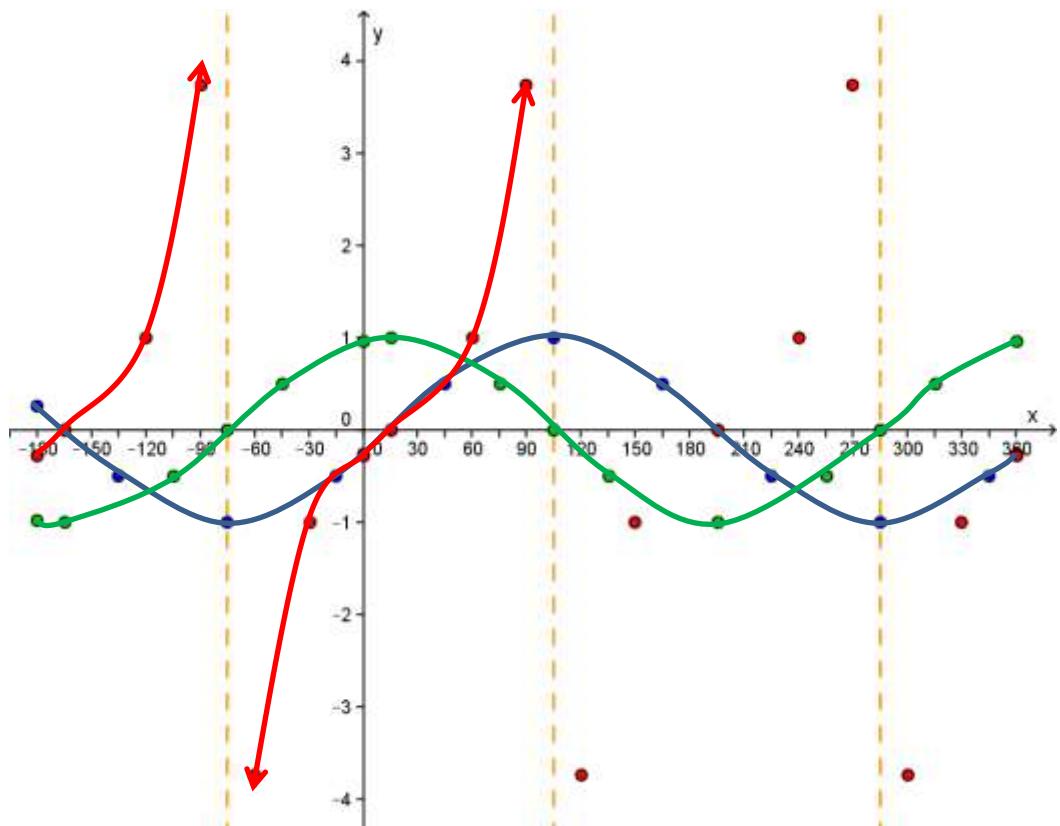
b) $x = -75;\quad x = 105\quad and\quad x = 285$

c)
$h(x) = \sin(x - 15)$	$j(x) = \cos(x - 15)$	$k(x) = \tan(x - 15)$
Period = 360	Period = 360	Period = 180
Amplitude = 1	Amplitude = 1	Amplitude = no amplitude

d) $x = -120;\quad x = 60;\quad x = 240$ \qquad (where $k(x) = 1$)

e) $x = -165;\quad x = 15;\quad x = 195;$ \qquad (where $k(x) = 0$)

f) where is $h(x) \geq j(x)$

$-180 < x \leq -120$ \qquad and \qquad $60 \leq x \leq 240$

g) Distance = 1